



Spiral Bevel Gears: Review on Dynamics

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ABSTRAK

Roda gigi bevel merupakan jenis roda gigi yang umum digunakan dalam berbagai aplikasi, terutama yang memerlukan perubahan arah transmisi daya. Gigi pada roda gigi bevel rentan terhadap berbagai cacat yang dapat memengaruhi kinerjanya, seperti pengelupasan, keausan, dan terkelupas. Cacat ini dapat menyebabkan peningkatan getaran, kebisingan, dan penurunan efisiensi sistem roda gigi. Untuk memastikan kinerja dan keawetan roda gigi bevel yang optimal, penting untuk mendiagnosis dan mengatasi cacat pada gigi dengan tepat. Makalah ini menyelidiki dinamika roda gigi bevel dalam berbagai kondisi operasional dan menganalisis dampak faktor-faktor, seperti modifikasi mahkota, pada kesalahan transmisi sistem. Penelitian ini mengeksplorasi konsep pembagian beban, merinci kondisi operasional dan model matematika untuk menghitung pembagian beban di antara planet. Selain itu, penelitian ini mengevaluasi faktor-faktor yang memengaruhi distribusi beban di antara planet, menekankan parameter seperti bantalan, serangan balik, kesalahan run-out, kesalahan posisi, dan kesalahan manufaktur sambil menilai efektivitas PGS dalam pembagian beban. Untuk meningkatkan pembagian beban, termasuk sistem mengambang dan pin fleksibel, metode dan solusi dibahas. PGS dikaji dari perspektif fase. Selain itu, makalah ini merangkum studi tentang dampak patahan pada fase planet.

ABSTRACT

Bevel gears are a type of gears commonly used in various applications, especially where a change in the direction of power transmission is needed. The teeth on bevel gears are vulnerable to various defects that can affect their performance, such as pitting, wear, and chipping. These defects can lead to increased vibrations, noise, and decreased efficiency of the gear system. In order to ensure optimal performance and longevity of bevel gears, it is essential to properly diagnose and address any defects on their teeth. The paper investigates the dynamics of bevel gears under varied operational conditions and analyzes the impact of factors, such as crowning modification, on system transmission error. This research explores load-sharing concepts, detailing operational conditions and mathematical models for calculating load-sharing among planets. Moreover, it evaluates factors influencing load distribution among planets, emphasizing parameters like bearing, backlash, run-out errors, position errors, and manufacturing errors while assessing PGS effectiveness in load-sharing. To enhance load-sharing, including floating systems and flexible pins, methods and solutions are discussed. PGS is examined from a phasing perspective. Additionally, this paper summarizes studies on the impact of faults on the planet phase.

1. Introduction

Classical usage of bevel gears traces back to the 3rd century B.C. with the adoption of 'Persian wheels' for lifting tasks [1]. Prior to the industrial era in the seventeenth century, belt drives sufficed for power transmission, but the rise of steam engines necessitated more efficient drives due to their increased power and rotational speeds. The nineteenth century witnessed the emergence of a gear manufacturing

industry, with Heinrich Schicht innovating the hobbing technique for cylindrical gears and adapting it for bevel gears by employing a conical hob to produce spiral bevel gears [2].

The manufacturing methods and design of bevel gears have continuously evolved to produce a variety of advanced configurations. Initially gaining prominence in the automotive

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industry in the early 20th century, bevel gears now play crucial roles in diverse applications including vehicle transmissions, aircraft engines, turbine systems, flap controls for aircraft wings, and marine propulsion systems.

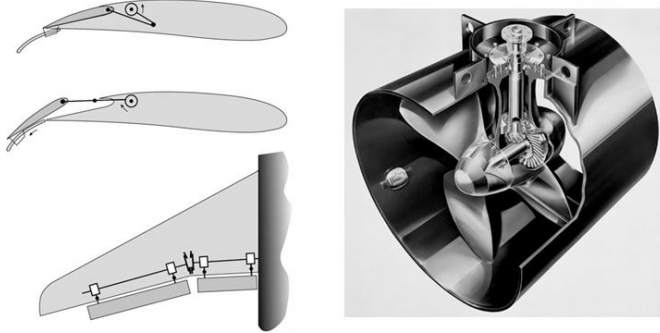


Fig. 1. Bevel gear application: a) vehicle transmissions, b) marine drives.

The paper presents research on the dynamics of bevel gears under diverse operational conditions and the impact of different factors, such as crowning modification, on the transmission error within the system. The sections of the paper are structured as follows: Section 2 delves into the concept of load-sharing, elucidating the various operational conditions employed by PGS. Additionally, this section introduces several mathematical models utilized by researchers to compute load-sharing among planets. Section 3 examines the factors influencing load distribution among planets and assesses the effectiveness of PGS in terms of load-sharing, highlighting parameters like bearing, backlash, run-out errors, position errors, and manufacturing errors. Section 4 explores different methods and solutions aimed at improving load-sharing, including floating systems and flexible pins. Section 5 focuses on investigating PGS from a phasing standpoint, while Section 6 provides a summary of studies conducted on the impact of faults on the planet phase.

2. Bevel gear classification

Bevel gears can be categorized based on various characteristics, including the variation of tooth depth across the face width, the tooth trace type, the form of the tooth trace curve, the pinion hypoid offset, the type of indexing operation, the cutting method, and the manufacturing technique [2-4]. Additionally, bevel gears can be classified according to their hypoid offset. Bevel gears without pinion offset have intersecting axes, while those with pinion offset, known as hypoid gears, have crossed axes. This paper specifically examines three commonly used types of bevel gears in applications: straight bevel gears, spiral bevel gears, and hypoid bevel gears.



Fig. 2. Bevel gear: a) straight bevel gear, b) spiral bevel gear, c) hypoid bevel gear.

2.1. Straight bevel gear

Straight bevel gears, the most basic type of conical gear, are both easy to manufacture and commonly used. Consequently, early investigations focused on this gear type. In 1980, Terauchi [5] conducted experimental research on the correlation between different speeds of straight bevel gears and torque fluctuations. Xiaoyuan et al. [6] studied the vibration characteristics of perforated straight bevel gears, which have holes on their webs.

In 1980, Terauchi [5] conducted experimental research to explore the relationship between different speeds of straight bevel gears and variations in torque, dynamic loads on gear teeth, and bending vibrations of the gear shaft under constant torque. Their investigation revealed a reciprocal influence between gear speed and dynamic load factor. They also observed that the dynamic load factor changes depending on the tooth trace location. Furthermore, they found that the natural frequency of torsional vibration in the gears affects shaft bending vibration, torque variation, and dynamic loads. In a subsequent study, the same authors [7] further investigated the dynamic behavior of straight bevel gears by analyzing gear vibration on rectangular coordinates. Their findings suggested that the stiffness of bevel gear shafts and resonance in torsional vibration affect displacement in horizontal, vertical, and axial directions of the gear. They demonstrated that reducing bearing stiffness leads to increased vibration amplitude. Xiaoyuan et al. [6] investigated the vibration characteristics of perforated straight bevel gears, which feature holes on their web, aiming to diminish noise and vibration within the gear system through gear modifications. Their experiments revealed that introducing holes in the gear enhances its flexibility, resulting in meshed gears with notably higher damping and reduced noise levels. Additionally, the perforated gear design was found to enhance stress distribution, decrease bending stress, and prolong gear lifespan.

Motahar et al. [8] investigated the nonlinear vibration of straight bevel gears by altering the tooth profile. In their study, three bevel gear models were examined: one featuring a

pure involute profile, another with a statically optimized tooth profile, and a third with a dynamically optimized tooth profile. Genetic algorithms were utilized to adjust the tooth profiles and determine the optimal amount and length of modifications. The dynamic responses obtained from a dynamic analyzer were compared both qualitatively and quantitatively. It was found that applying tooth profile modifications to both statically and dynamically optimized gear pairs significantly reduced the average dynamic response. Furthermore, the presence of period was eliminated in both optimized simulations through the implementation of tooth optimization.

Han et al. [9] introduced a dynamic model for a coupled bevel gear pair and planetary gear train system utilizing the hybrid lumped parameter approach. They identified an amplitude modulation phenomenon in the time-domain signal of the intact coupled system. For practical purposes, they advised against positioning the support bearings too closely to both ends of the intermediate shaft during the design of supporting bearings to enhance system functionality and overall performance.

2.2. Spiral bevel gear

The spiral bevel gear represents the most intricate type of bevel gear, often employed in applications necessitating substantial load capacity at elevated operating speeds beyond what straight bevel gears can typically withstand. The correlation between spiral and straight bevel gears mirrors that of helical and spur gears.

The outstanding merits of spiral bevel gears (SBGs), including quiet operation, adaptability to housing distortion, high efficiency, wide adjustment range, and durability, have made them highly favored among automobile and axle designers. However, the primary drawback of this gear type is the thrust loading it imposes. In 1915, Stewart [10] emphasized the advantages of SBGs in automobile drives compared to other gear types. Through a comparison of straight and spiral bevel gears, Stewart noted that spiral teeth experience greater loads than straight teeth.

In their research, He et al. [11] performed a dynamic analysis of a bucket elevator system incorporating both spiral bevel gears (SBG) and a planetary gear set. They constructed a dynamic model using the lumped mass method, considering the time-varying mesh stiffness. The team examined the impacts of different input rotational speeds and power levels on the system's dynamic behavior. Their results showed a gradual decrease in dynamic mesh force with rising speed, while it increased with higher power input. Additionally, they noted that in low-speed and heavy-load transmission setups, power exerted a more pronounced effect on the mesh force.

Peng and Lim [12] formulated an extensive dynamic model for a hypoid geared rotor system, accounting for multiple degrees of freedom and bodies. Their study underscored the importance of the pinion or gear body shape and its influence on the ratio between the bending (pitch and yaw) moment of inertia and the torsional (roll) moment of inertia, which directly impacts the gyroscopic effect. Interestingly, they discovered that the gyroscopic effect becomes negligible at low operating speeds or when the gear pair's out-of-phase

torsion vibration dominates the peak response. Moreover, their investigation revealed that incorporating gyroscopic terms in nonlinear time-varying simulations, particularly during non-stationary or transient operations, produced results akin to those derived from linear time-invariant analysis.

Numerous studies on spiral bevel gears (SBGs) have concentrated on tooth contact analysis (TCA) to identify the primary sources of vibration. Gabiccini et al. [13] introduced an automated procedure for optimizing the loaded tooth contact pattern of face-milled hypoid gears, considering variations in misalignments within specified ranges. They proposed a two-step methodology to address the issue: first, modifying the microtopography of pinion teeth to match a target area while ensuring clearance from the edges; and second, identifying a subset of machine-tool settings to achieve the necessary topography adjustments. The optimization challenges were formulated and resolved as unconstrained nonlinear problems, taking into account robustness concerns regarding misalignment variations. They demonstrated that this direct optimization approach can design hypoid gear pairs that are both satisfactory under nominal conditions and globally robust, thereby circumventing trial-and-error iterations.

3. Methodology

Computerized methodologies for assessing dynamic loads and analyzing surface temperature and film thickness at the contact interface between pinion and gear teeth were outlined in studies by [14] and [15]. These analyses enhance the predictive capability for lubrication-related failures. Experimental investigations on spiral bevel gears (SBGs) carried out at NASA Lewis Research Center revealed significant variations in dynamic loads with gear shaft speed, correlating with critical frequencies of the system. The film thickness along the contact path remained relatively uniform, primarily affected by entrainment velocity and changes in bulk surface temperature.

In an independent investigation conducted by Fan et al., a dynamic model was formulated for a system comprising coupled rotors with a radial driveshaft [16]. They simulated the mode shapes and unbalanced responses of this system, with particular emphasis on introducing the concept of coupled stiffness matrix and coupled damping matrix for the spiral bevel gear (SBG) pair. Through an analysis of lateral vibration response to imbalance, they identified the natural frequencies and corresponding shapes of the system. Li conducted a study on the dynamic characteristics of angled bevel geared systems under the influence of non-linear oil film forces [17]. Xia, as outlined in [18], presented an MSc thesis investigating the impact of gear-shaft-bearing structural design on the dynamics of spiral and hypoid bevel gear systems. He developed new computationally precise models for the dynamic characteristics of gear-shaft-bearings, which could be utilized to forecast the dynamic response of the spiral bevel gear (SBG) rotor system. In general, he concluded that transitioning from a 2-bearing configuration to a 3-bearing configuration significantly influences the mesh and dynamics of SBGs. In the study conducted by Yang and Zhang [19], a reduction in transmission error (TE) was achieved for both

standard and modified straight bevel gears (SBGs). The findings indicated that the modified SBG pair exhibited superior dynamic performance compared to the standard version. This outcome confirmed the effectiveness of the novel approach involving the modification of tooth faces using specialized tools in reducing gear vibration and noise.

Roulois et al. [20] employed a global method to calculate the noise generated by spiral bevel gearboxes in helicopters, aiming to gain a better understanding of the vibroacoustic behavior of operating gearboxes. Peng's doctoral dissertation [21] employed multi-body and multi-degree-of-freedom (MDOF) models to investigate the dynamic behavior and vibration of hypoid and bevel geared rotor systems. He examined the effects of gear geometric eccentricity and shaft assembly error on spiral bevel gears (SBG) meshing, discovering differences in error effects and sensitivities compared to hypoid gears. In contrast, Wilson et al. [22] focused on analyzing techniques for SBGs, utilizing the finite element method (FEM) to streamline analysis equations. Their efforts aimed to enhance SBG performance in terms of noise, vibration, durability, and efficiency. Additionally, they integrated simulations with hardware testing, prototype fabrication, and reduced test samples to propose more accurate design characteristics.

Lin et al. [23] investigated the dynamic characteristics of gearboxes incorporating SBGs used in ship lifting systems. They found that displacement processes inevitably generate vibration and noise due to dynamic excitation. To analyze the system's vibration response, they developed a dynamic finite element model combined with gearbox transmission and housing structure models, ensuring minimal interference of system frequencies with rotation. Astoul et al. [24] introduced a novel methodology to reduce transmission error (TE) in SBGs. Drawing on NASA findings, they attributed helicopter gearbox noise primarily to gear meshing, which increases with power transmission. Their optimization method relied on the relationships between maximum contact pressure, contact ratio, load sharing, and quasi-static TE, predicting the initial contact pattern between gears to reduce TE. Junfeng et al. [25] devised a nonlinear dynamic model for bevel gear transmission systems, considering factors like time-varying mesh stiffness. Their work provided a theoretical foundation and methodology for designing lightweight, high-performance gear systems, revealing various steady-state response patterns with changes in excitation frequency. Xu et al. [26] explored the bending-torsional vibrations of SBG systems, analyzing factors including gear backlash, static transmission error, time-varying stiffness, and bearing distances. They found that increasing the damping ratio reduces the size of the disturbed area and examined parameters like the load factor and gear return's impact on system behavior. Feng et al. [27] investigated the influence of geometry and design on SBG strength and dynamic behavior, emphasizing design parameters such as pressure angle, helix angle, root fillet radius, and tooth thickness ratio. Optimization within suitable vibration frequency ranges enhanced gear performance and longevity.

Yang et al. [28] present a novel method to enhance the transmission efficiency of spiral bevel and hypoid gears by implementing gear surface modifications through curvature

synthesis. Their numerical simulations and analysis illuminate the dynamic behavior of hypoid gear pairs with varying degrees of transmission errors (TEs), emphasizing the impact of TE characteristics on the gear system's performance and acoustic properties. The dynamic system shown in Fig. 3 is a 14-DOF lumped parameter model.

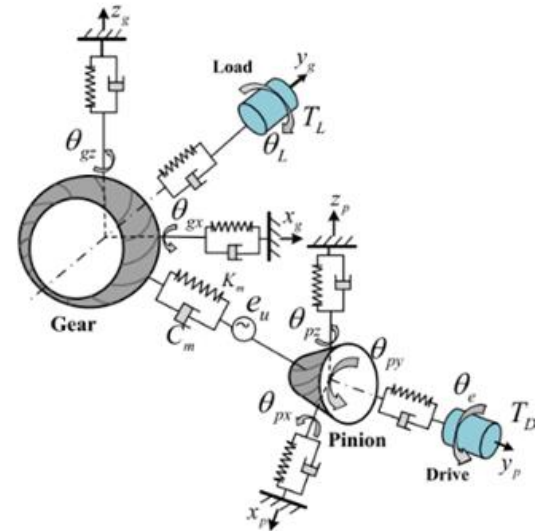


Fig. 3. Lumped parameter model of hypoid geared rotor system, [28]

Yavuz et al. proposed a model for a spiral bevel gear (SBG) train that incorporates factors such as flexible shafts, bearings, gear backlash, and gear stiffness [29]. Their study utilized a combination of finite element and discrete mesh models to develop a dynamic model of the system and employed mathematical methods to predict system behavior under various conditions. They validated the model's accuracy by comparing results obtained through mathematical methods with those from direct numerical integration. Additionally, they investigated the effects of factors like backlash and static mesh force on system behavior.

Another study proposed the development of a dynamic model for a drivetrain with an SBG pair, including shaft bearings and gear backlash flexibilities [30]. The model was solved using the harmonic balance method and verified through direct numerical integration. The study explored various factors affecting the system's dynamics and compared results with a gear torsional model. Zhu et al. investigated the dynamic characteristics of an SBG transmission system, finding that factors such as shaft angle, gearbox flexibility, and unbalance influence system vibration [31]. Their study establishes a theoretical foundation for optimizing dynamic performance and mitigating vibration and noise.

Chen et al. described a new mathematical model and numerical strategy for calculating the performance of elastic ring squeeze film dampers (ERSFDs) in rotating machinery, particularly SBG drives [32]. They evaluated the performance of the proposed model and demonstrated that ERSFDs outperform classical squeeze film dampers (SFDs) in suppressing nonlinear characteristics within specific speed ranges. Lafi et al. presented an analytical model for analyzing the dynamic behavior of a differential bevel gear set in

automobiles, considering flexible bearings and gear mesh stiffness [33]. Their study incorporated the position error of the differential pinion and compared results with a healthy system to provide insights for improving gear set design.

Sun et al. discussed factors affecting the calculation method and results of the dynamic mesh force in SBGs [34]. They compared different mesh models and approaches for estimating mesh stiffness representations to identify suitable methods for estimating dynamic meshing loads. Hua and Chen studied the dynamic responses of hypoid and spiral bevel geared systems, focusing on the impact of bearing elasticity [38]. Their investigation aimed to provide insights into how bearing stiffness affects the dynamic behavior of such systems. Hou et al. employed a vector form intrinsic finite element method (FEM) and a novel damping model to investigate SBG dynamics [41]. Their proposed method, incorporating a damping force proportional to relative velocity, showed excellent convergence and efficiency for large-scale, high-speed simulations.

Lu et al. proposed a semi-analytical model for dynamic analysis of thin-walled SBG transmissions under high-speed, heavy-load conditions [42]. Their model accounted for the influence of applied load on contact behavior, offering accurate mechanical deformations for different parts of the gear structure. Ding et al. developed a semi-finite element method to predict the transmission behavior of SBGs under high speed and heavy loads [44]. Their method accurately forecasts dynamic impacts' dimensions and positions, facilitating precise analysis of SBG transmission dynamics. Liu et al. developed a parametric modeling approach to predict the vibration response of high-speed gear transmission systems [45]. Their method effectively characterizes frequency and average errors in time and frequency domains, aiding in more efficient transmission system design.

Tian et al. employed the component modal synthesis method to model squeeze film dampers in SBGs, considering factors like time-varying mesh stiffness and gyroscopic effects [46]. Their study demonstrates the effectiveness of integrating squeeze film dampers in reducing rotor motion amplitude during critical speed crossings and enhancing transmission efficiency in SBG systems. Jorani et al. [47] employed vibration analysis techniques and statistical process control charts to detect gear cracks. Their innovative approach combines statistical process control charts, time domain analysis, and frequency domain analysis within a 10-degree-of-freedom dynamic model of spiral bevel gear (SBG) systems. By monitoring vibration states, this non-destructive technique effectively identifies teeth propagating cracks in gear systems. Results indicated that the exponentially weighted moving average chart surpassed frequency domain analysis, time domain analysis, and the Shewhart X-bar chart in detecting early-stage cracks across all levels.

3. Manufacturing

Fuentes et al. devised an integrated computerized method for designing and assessing low-noise spiral bevel gear (SBG) drives with adjusted bearing contact. This approach comprises four distinct computational steps. Firstly, they

establish a parabolic function to mitigate alignment errors and minimize the bearing contact shift induced by misalignment. Next, finite element analysis is employed to ascertain contact and bending stresses and examine bearing contact formation. The design of finite element models and boundary conditions is automated, eliminating the need for intermediate CAD software. Stress analysis is carried out using a commercially available finite element analysis program equipped with contact capability. The efficacy of the developed theory is illustrated through numerical examples [48].

Yinong et al. investigated the effects of uneven mesh stiffness on the vibration behavior of spiral bevel gear (SBG) transmission systems. Their results suggest that the mesh stiffness on the drive side exerts a more pronounced influence on the dynamic response compared to the coast side. Furthermore, they observed that modifying the mesh stiffness on the drive side impacts the response across all excitation frequency ranges. Moreover, they noted that vibration characteristics exhibit significant improvement under heavily loaded conditions compared to lightly loaded ones. Additionally, they highlighted that asymmetry in mesh stiffness introduces additional effects on the dynamic response, particularly beyond what is observed in heavily loaded scenarios [49].

Yang et al. [50] conducted a study highlighting the substantial influence of machining processes on the dynamic stability of spiral bevel gears (SBGs), affecting factors such as excited amplitude within the system. The researchers concluded that coating the gears through an ultrasonic method proves effective in enhancing gear transmission quality. The ultrasonic gear coating process demonstrates non-smooth dynamic characteristics with significant nonlinear properties. Through careful selection of machining parameters and utilization of the ultrasonic method, high-quality gears can be obtained. This approach not only minimizes the transmission error range but also enhances the overall dynamic performance of the gears.

Miaoxin et al. [51] investigated the grinding process of spiral bevel gears (SBGs) and its impact on system vibration. The findings revealed that when SBGs are being ground and the grinding tool directly engages with the teeth, a distinct step jump occurs. Moreover, increasing the torque to a certain level results in a different type of step jump. The most optimal meshing mode for SBGs under this torque occurs when the gear does not experience a sudden impact.

In their research, Lee et al. [52] evaluated the influence of misalignment on the tooth profile geometry of SBGs. Addressing concerns regarding inconsistency, they proposed modifications to the tooth profile and the pinion arch. By adjusting the pinion arch and dedendum, they effectively lowered the natural frequencies of the gear drives. Implementation of the modified tooth profile resulted in the elimination of critical dynamic behavior in the gear drives.

Gonzalez et al. [53] delved into the determination of initial machine tool settings for spiral bevel gear (SBG) production and the compensation of alignment errors in the cyclopid system. Face hobbing is a prevalent method in industry for SBG production. Localizing bearing contact is a common

practice in SBG drives. Their study involved computer-generated tooth surface creation for an SBG drive considering the cyclopid system and establishing kinematic conditions to control tooth thickness. It is imperative to determine the initial machine tool settings from the initial transfer data for SBGs generated via the cyclo-payload system.

Cai et al. [54] conducted an analysis of parameters in a double circular arc SBG drive system, focusing on calculating transmission efficiency while considering helix angle, cone distance, and friction coefficient. Highlighting the importance of system efficiency, the analysis revealed that an optimal nutation angle falls within the range of 0° to 15° . Similarly, the recommended range for the spiral angle was found to be 10° to 25° . The study further indicated that the system's efficiency is minimally affected by the taper distance, whereas the coefficient of friction exerts a significantly greater influence.

4. Resonances

Analytical determination of resonant frequencies for the RAH-66 helicopter fantail gears was conducted by [55] with the aim of achieving a gear design devoid of damaging resonant responses within the operational speed range. The proposed analytical method aimed to enhance vibrational responses, favoring a criterion based on stress levels over a deflection ratio. Their method involved loads rotating relative to the gear, allowing for the incorporation of forward and backward traveling wave excitation effects in computed dynamic responses.

In their study, Peng et al. [56] presented an analysis of SBGs, emphasizing their sensitivity to eccentricity. Employing the SBGs rotor system model for eccentricity modeling and dynamic analysis, they conducted frequency spectrum analysis revealing significant sideband responses. These responses, stemming from modulation behavior induced by eccentricity, could be accurately simulated using existing models. Notably, larger eccentricity or system resonance at higher shaft orders intensified sideband responses.

When designing gears, it's essential to consider the noise generated by TE. Wang et al. [57] investigated this issue and found that dynamic mesh forces and tooth pair friction are pivotal in gear dynamics. They noted that the interaction between gear pairs and their gyroscopic effects profoundly influences resonance within specific frequency ranges. The dynamic forces experienced by bearings, both horizontally and vertically, are highly sensitive to dynamic mesh force and gyroscopic effects. To reduce vibration responses and gear noise, increasing the pinion's inertia and enhancing pinion bearing stiffness are recommended.

Additionally, a study on the dynamic characteristics of an ultrasonic vibration system used in processing SBGs is discussed [58]. The researchers developed a mathematical model of resonance for the gear's ultrasonic vibration system and analyzed its frequency and displacement characteristics. Through experiments on various hypoid gears, they observed significant influences of the ultrasonic vibration system's dynamic characteristics on the finished machining quality of the gear. This study underscores the importance of considering gear characteristic parameters when designing an ultrasonic vibration system for processing spiral bevel gears.

In the realm of high-speed gear dynamics, Shi et al. [59] delved into the significant concerns surrounding noise and vibration. They observed that the substantial amplitude of dynamic contact forces can precipitate premature failures such as wear and pitting, thereby impacting dynamic responses. Their investigation revealed that, compared to quasi-static scenarios, dynamic fluctuations result in heightened vibration amplitudes at resonance peaks.

Furthermore, Luan et al. [60] embarked on an experimental and numerical exploration of stress distribution characteristics in high-speed SBGs. Their findings unveiled the potential for gear failure stemming from traveling wave resonance, with possible catastrophic consequences, particularly in aero-engines. Through a combination of experiments and simulations, they scrutinized the stress distribution traits of high-speed SBGs within aero-engines. By analyzing pressure and speed graphs, they pinpointed the time of fracture and maximum stress at specific positions within the prefabricated gear. The results suggested that this gear could endure operation within the resonance range without fracturing.

5. Bearing Stiffnesses

Hua et al. [61] emphasized the necessity of considering component flexibility, such as bearings and shafts, to accurately predict the kinematic behavior of SBGs. They underscored the importance of bearing configuration on SBG engagement and kinematics, noting substantial changes in pinion shaft-bearing stiffness when transitioning from a 2-bearing model to a 3-bearing model. Horizontal and vertical translational stiffness, along with bending stiffness, exhibited notable increases, while the correlation between translational and bending stiffness decreased.

Similarly, Jinli et al. [62] highlighted the significant impact of the shaft, a pivotal parameter in a drive system, on bearing stiffness. Their research elucidated that dynamic mesh force resulting from TEs is the primary cause of vibration within a transmission system. Furthermore, they observed that enhancing bearing stiffness effectively mitigates vibration response at high speeds. However, a significant increase in bearing stiffness may compromise its ability to dampen vibration response. Consequently, the resonance regions of the coupled system shift relative to the bearing stiffness at different vibration response speeds. In essence, Jinli et al. underscored the interplay between shaft properties, bearing stiffness, and vibration characteristics in a drive system.

In their research, Baron et al. [63] investigated the operational performance of a stationary industrial gearbox by analyzing dynamic signals extracted from the pinion bearing housing. Bearings play a pivotal role in power transmission systems, comprising various elements such as shafts, gears, and bearings themselves. Among these components, bearings stand out as the second most common source of vibration due to wear or damage. To forestall defects, corrective and preventive measures are implemented, including vibration detection, tribology, thermal detection, and microscopic analysis, all aimed at pinpointing the root causes of failure. Ensuring the optimal operation of bearings is paramount for attaining prolonged life and reliability of the gearbox.

Precisely evaluating wear entails determining the excitation frequencies of specific bearing parts, including the inner and outer rings, cage, and rolling elements.

Wang et al. [64] conducted a study on the impact of component flexibility on system dynamics. They observed that the vibration energy generated by transmission error (TE) in the gears propagates to the outer housing via the shafts and bearings. Initially, the research focused on evaluating the influence of shaft elasticity on TE by comparing a gear pair model with an axis system model. The findings revealed that TE is highly responsive to system tension, and the stiffness of the bearings significantly shapes the characteristics of the gear system.

Mabrouk et al. [65] concentrated on the dynamic vibration of a vertical axis wind turbine. They noted that the turbine exhibits inherent instability and aerodynamic behavior due to the continuous variation in blade angle relative to its original position during rotation. To deepen the understanding of this wind turbine's dynamic behavior, the researchers introduced a novel method facilitating the exploration of complex aerodynamic phenomena alongside the mechanical gear system's vibration. This approach enables the prediction and analysis of the turbine's intricate aerodynamic behavior.

6. Nonlinear vibration

Litvin's seminal study on spiral bevel gears (SBGs) was presented in 1983 [66]. His approach centered on optimizing SBG geometry to attain superior gear design. Litvin established relationships between the directions of the tooth surface and the principal curvatures of SBG teeth to compute reliability and lifespan for both helical and spur gears. In the same year, Satoshi et al. [67] conducted measurements of radial, axial, and circumferential accelerations, along with dynamic loads of Gleason-type straight-bevel gears, to investigate their dynamic behavior.

Jiang et al. analyzed the lateral, torsional, and axial vibrations of a rotor-bearing system with bevel gears [68]. They discovered interconnections among lateral, torsional, and axial vibrations, necessitating a coupled analysis. In their subsequent work [69], they delved deeper into the dynamics of coupled lateral, torsional, and axial vibration in such systems. Developing a dynamic model for perpendicular spiral bevel-gear rotor-bearing systems, they scrutinized system dynamics encompassing unbalanced responses, critical speeds, and stability threshold speeds. Their observations underscored the significant impact of boundary conditions compared to rigid supports. The dynamic model is shown in Fig. 3.

Suslin and Pilla [70] conducted an analysis on the dynamic loads of bevel gears with circular and straight teeth, examining the benefits of spiral bevel gears (SBGs) with an overlap ratio exceeding two. They explored the effects of setup errors, mesh stiffness, and overlap ratio on the gears' dynamic factor. They proposed that a lower dynamic factor could potentially enhance the fatigue life of the gears, while slight variations in the overlap ratio might mitigate the risk of torsional vibrations. However, longevity tests were not conducted.

Mabrouk and Hami utilized numerical simulations to investigate the behavior of a Darrieus wind turbine under unsteady wind conditions. They observed significant impacts of changes in inlet velocity on the turbine's aerodynamic performance and vibration, offering insights for Vertical Axis Wind Turbines design. Furthermore, the researchers examined the influence of design parameters on the vibration of the turbine's geared transmission system. Specifically, they explored the relationship between aerodynamic performance and dynamic vibration, aiming to establish a correlation between the two. Their findings highlighted the notable influence of the number of rotor blades on both turbine efficiency and dynamic response [71, 72].

A finite element model of the gear shaft and a mechanical model of the spiral bevel gear (SBG) were established to investigate the dynamic characteristics of a helicopter tail transmission system [73]. The analysis included studying the effects of unbalanced excitation, transmission error (TE), gearbox flexibility, time-varying stiffness, and shaft and bearing support on system vibration, offering insights to mitigate noise and vibration and optimize dynamic performance.

An innovative approach known as ease-off flank modification was proposed in [74] and [75] to address running vibration issues in SBGs. This technique effectively minimized loaded tooth-to-tooth error (for low-speed SBGs) and meshing impact (for high-speed SBGs). The researchers concluded that the novel ease-off flank modification significantly reduces running vibration and enhances the dynamic performance of high-speed, high contact ratio SBGs. In essence, they introduced a new method to alter the shape of gear teeth in SBGs, resulting in reduced vibration during operation and overall improved gear performance.

Wang et al., [76], [77], employed a finite element method (FEM) to model the nonlinear vibration of a spatial geared

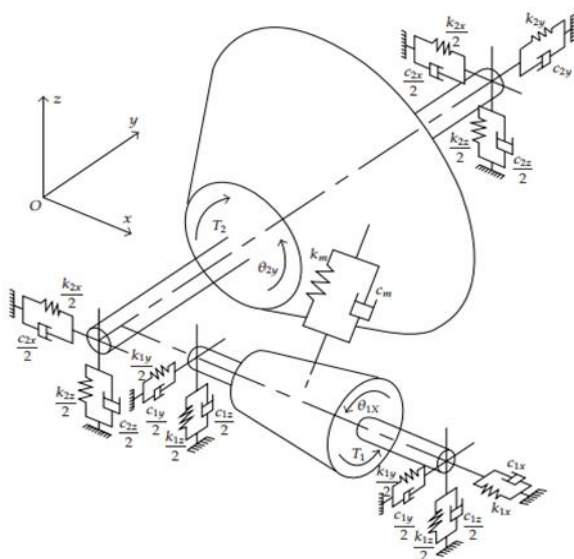


Fig. 3. A dynamic model of spiral bevel gear, [68].

system with intersecting axes. They demonstrated that the lateral stiffness of the shaft significantly influences the resonance peak frequencies of the vibration. Increasing the lateral stiffness leads to subsequent increases in resonance peak frequencies. Furthermore, they illustrated that the torsional stiffness of the shaft primarily impacts the magnitude of the dynamic load and vibration. Decreasing torsional stiffness results in lower vibration levels. Additionally, they devised a method based on specific finite element theory to depict the dynamic behaviors of spatial systems concerning the stiffness of the shaft, gear, and link. An 8-degree-of-freedom (DOF) dynamic model, incorporating time-varying stiffness and backlash, was discussed to analyze the nonlinear frequency response characteristics of an SBG system. The study revealed that the occurrence of the jump phenomenon is consistent across different supporting systems [78].

Karai et al. [79] conducted a study on the dynamic response of a single-stage bevel gear under the influence of localized damage. The quest to minimize noise in gearboxes and gears has become a significant focus in the industrial sector, aiming to stabilize vibrations resulting from power transmission within acceptable thresholds. Through various simulations, the researchers compared power transmission in both intact and damaged gears. The damage was simulated by locally reducing the meshing stiffness to mimic cracking damage. Their findings highlighted the pivotal role of momentum in exacerbating vibration levels.

Samani et al. aimed to investigate the vibration of nonlinear SBGs using an innovative approach of tooth surface modification [80]. They explored the nonlinear vibration of a novel method designed to enhance SBGs with higher-order transmission error (HTE) and discovered that HTE can refine load sharing and reduce tensile stress at the tooth root. Consequently, the meshing quality of HTE spiral bevel gears appears more suitable than that of conventional meshing quality gears. However, efforts to develop dynamic vibration analysis, particularly regarding super-harmonic resonances, were unsuccessful. Liu et al. [81] conducted a study on the nonlinear dynamic analysis of the central bevel gear transmission system in an aero engine, considering both internal and external excitations. They formulated a model using lumped parameter models to explore the system's nonlinear dynamic behavior. The model integrated internal excitations like time-varying stiffness, gear backlash, and tooth-to-tooth errors, along with external factors such as input power, speed variations of the driven shaft, and rotor unbalance. The findings suggested that effective management of input power, control of speed fluctuations, and mitigation of high-pressure rotor mass imbalance could notably enhance the vibration characteristics of the system.

Wu et al. [82] proposed the modulation signal bi-spectrum (MSB) as a monitoring technique with high sensitivity to gear tooth defects. They experimentally validated the effectiveness of this approach using vibration data obtained from a run-to-failure test. The results illustrated that MSB is a suitable method for monitoring the wear process and implementing running control of SBGs during their early stages of operation. The dynamic behavior of SBGs with surface defects was explored using a transient mixed lubrication model along

with a two-degree-of-freedom torsional dynamic model, as discussed in [83]. The study unveiled that surface defects significantly influence the meshing process of SBGs. Additionally, the dynamic response of the gears varied based on the position and geometry of the defect. In essence, [83] scrutinized the impact of surface defects on the dynamic behavior of SBGs, emphasizing the importance of defect position and geometry on both the meshing process and resulting dynamic response.

Summary of spiral bevel gear vibrations

Section	Main key points	Author (Year) [References]
Bevel gear classification	Bevel gear	Gupta, K., Jain, N. K., & Laubscher, R. (2017). [1]; Klingelnberg, J. (2016) [2], [9];
	Nonlinear dynamic	Chen, S., Tang, J., Chen, W., Hu, Z., & Cao, M. (2014) [3];
	Dynamic of bevel gear	Terauchi, Y., Miyao, Y., Fujii, M., & Sagawa, K. (1980) [4]; Terauchi, Y., Miyao, Y., Fujii, M., & Sagawa, K. (1980b) [5]; Terauchi, Y., Fujii, M., & Hoto, H. (1981) [7]; He, Z., Xing, Z., Zhou, Q., & Chang, L. (2023) [11]; Peng, T., & Lim, T. C. (2009) [12];
	Vibration	Z. Xiaoyuan, F. Zongde, and L. Wei [6]; Motahar, H., Samani, F. S., & Molaie, M. (2015) [8];
	Contact pattern	Gabiccini, M., Bracci, A., & Guiggiani, M. (2010) [13];

Section	Main key points	Author (Year) [References]
Methodology	Computer solution	Chao, H. C., & Cheng, H. S. (1987) [14]; Hsu, C. Y., & Cheng, H. S. (1987) [15]; Fan, Y. S., Wang, S. M., & Yang, Z. (2008) [16];
	Dynamic modelling	Lafi, W., Djemal, F., Tounsi, D., Akrouit, A., Walha, L., & Haddar, M. (2019) [33]; Lu, S., Ding, H., Rong, K., Rong, S.,

Methodology	Nonlinear dynamic	Tang, J., & Xing, B. (2022) [42]; Tian, Z., Hu, Z., Tang, J., Chen, S., Kong, X., Wang, Z., Zhang, J., & Ding, H. (2023) [46]; Li,M.(2008) [17]; Yavuz, S. D., Saribay, Z. B., & Cigeroglu, E. (2017) [29]; Yavuz, S. D., Saribay, Z. B., & Cigeroglu, E. (2018) [30]; Chen, W., Chen, S., Hu, Z., Tang, J., & Li, H. (2019) [32]; Yavuz, S. D., Saribay, Z. B., & Cigeroglu, E. (2020) [35]; Chen, W., Chen, S., Tang, J., & Li, H. (2020) [37]; X. Hua(2010) [18] T. Peng(2010) [21]; Lin, T. J., He, Z. Y., Geng, F. Y., & Que, H. J. (2013) [23]; Feng, Z., & Song, C. (2017) [27]; Zhu, H., Chen, W., Zhu, R., Gao, J., & Liao, M. (2019) [31]; X. Hua and Z. Chen (2020, July 7) [38]; Lafi, W., Djemal, F., Akrouit, A., Walha, L., & Haddar, M. (2021) [40]; X. Hou, Y. Zhang, H. Zhang, J. Zhang, Z. Li, and R. Zhu [41]; Cao, X., Lu, F., Bao, H., & Zhu, R. (2022) [43]; H. Yang and Y. Zhang [19]; G. Roulois, F. Marrot, J. Caillet, A. Lored, and T. Dupont [20]; Wilson, B. K., Peterson, G., Kulkarni, A., & Kanase, A. (2011) [22]; Astoul, J., Mermoz, E., Sartor, M., Linares, J., & Bernard, A. (2014) [24]; Yang, J., Shi, Z., Zhang, H., Li, T., Nie,	Vibration	S., & Wei, B. (2018) [28]; Ding, H., Rong, S., Rong, K., & Tang, J. (2022) [44]; Liu, Z., Wei, H., Wei, J., Xu, Z., & Liu, Y. (2023) [45]; Lafi, W., Djemal, F., Tounsi, D., Akrouit, A., Walha, L., & Haddar, M. (2019) [34]; Chen, W., Chen, S., Hu, Z., Tang, J., & Li, H. (2020) [36]; Hua, X., Qiang, S., & Hang, W. (2020) [39]; Jorani, R. M., Haddar, M., Chaari, F., & Haddar, M. (2023) [47]; Junfeng, N., Guangbin, Y., Ye, S., Zhigang, Q., Minli, Z., & Xingfu, Z. (2016) [25]; Xu, J., Zeng, F., & Su, X. (2017) [26]; Litvin, F. L., & Coy, J. J. (1983) [48];
		Nonlinear vibration		
		Spiral bevel geometry		

Manufacturing	Dynamic behavior	Oda, S., Koide, T., & Okamura, Y. (1983) [67]; Li, Z., Wang, J., & Zhu, R. (2016) [52];	Nonlinear dynamics Experimental	Shi, Z., & Li, S. (2022) [59]; Luan, X., Gao, Y., Zhang, Z., Sha, Y., & Liu, G. (2023) [60];
		Mu, Y., He, X., & Fang, Z. (2021) [74]; Li, M., Hu, H., Jiang, P., & Yu, L. (2002) [68]; Li, M., & Hu, H. (2003) [69];		
	Vibration	Xiao, M. X., & Yang, J. J. (2014) [51]; Suslin, A. V., & Pilla, C. K. (2018) [70];		
		Mu, Y., He, X., & Fang, Z. (2021a) [75];		
	Transmission error (TE)	Fuentes, A., Litvin, F. L., Mullins, B. R., Woods, R., & Handschuh, R. F. (2002) [48]; Yinong, L., Guiyan, L., & Ling, Z. (2010) [49];		
		Cai, Y., Yao, L., Xie, Z., Zhang, J., & Peng, C. (2017) [54];		
Ultrasonic method	Mabrouk, I. B., & Hami, A. E. (2019) [71]; Mabrouk, I. B., & Hami, A. E. (2019b) [72];			
	Alsmus, M. (2019) [73];			
Alignment error	Yang, J. J., Wei, B. Y., Deng, X. Z., & De Fang, Z. (2011) [50]			
	Gonzalez-Perez, I., & Fuentes-Aznar, A. (2017) [53];			

Resonances	Dynamic	Grugel, R., Kim, S., Woodward, T., & Wang, T. (1992) [55]; T. Peng, T. C. Lim, and J. Yang [56];
		Y. Wang, J. Yang, D. Guo, and T. C. Lim [57];
	Vibration	Zhu, D., Yang, J., Deng, X., Jiang, C., & Li, J. (2018) [58];
Kinematic behavior		Hua, X., Lim, T., & Peng, T. (2011) [61];

Section	Main key points	Author (Year) [References]
Bearing Stiffnesses	Nonlinear dynamic	J. Xu, W. Lei, and W. Luo [62]; Baron, P., Kočiško, M., Blaško, L., & Szentivanyi, P. (2017) [63]; Wang, Y., Li, X., Qiao, G., & Lim, T. (2017) [64]; Mabrouk, I. B., Hami, A. E., Walha, L., Zghal, B., & Haddar, M. (2017) [65];
	Analysis of dynamic	
	Transmission error (TE)	
	Vibration	

Section	Main key points	Author (Year) [References]
Nonlinear vibration	Nonlinear dynamic	Wang, Y., Cheung, H. M. E., & Zhang, W. J. (2001) [76]; Wang, Y., Zhang, W., & Cheung, H. (2001) [77]; Wang S-m, Shen Y, and Dong H. [78]; J. Liu, H. Zhang, J. Zhai, and Q. Han [81]; Samani, F. S., Molaie, M., & Pellicano, F. (2019) [80]; Karray, M., Chaari, F., Viadero, F., Del Rincon, A. F., & Haddar, M. (2012) [79]; Wu, Z., Gu, F., Wang, T., Zhang, R., Shi, Y., & Ball, A. D. (2020) [82]; Pei, X., Pu, W., & Wang, Z. (2021) [83].
	Nonlinear vibration	
	Transmission error (TE)	
	Experimental	
	Dynamic behavior	

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